Velocity shear impoundment of the Io plasma torus

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Abstract. The Io plasma torus apparently has a much longer lifetime than anticipated on the basis of simple theoretical stability analyses. This quandary is particularly evident at the outer edge of the torus, the plasma ramp, where the steep radial gradient in plasma content indicates the presence of some confining influence. Previous attempts to explain this feature have focused on the possibility that high-energy particles could impound the colder, Iogenic plasma, but these studies have proved inconclusive. We consider an alternative mechanism whereby the development of an unstable perturbation is interrupted by the observed shear in the rotational velocity. An example in simplified geometry demonstrates how the relative azimuthal displacement of radially adjacent perturbations might eliminate their coherency and impose a finite saturation amplitude. Fully nonlinear numerical simulations using the Rice convection model at Jupiter produce analogous results, suggesting that perturbations are suppressed where the shear is strong. The crucial parameter is the ratio of the classic, linear growth rate to a rate that characterizes the velocity shear. The electric fields produced by unstable perturbations farther out are effectively shielded from the shear region. We suggest that this effect helps impound the plasma torus and is at least partly responsible for producing the ramp.

1. Introduction and Background

The discovery of the Io plasma torus was one of the outstanding achievements of the Voyager mission. Continually renewed at a ton every second, its million tons of total mass dwarfs that in the terrestrial magnetosphere. Although encircling Jupiter at a distance comparable to the geocentric distance of our own Moon, the plasma completes a rotation every 10 hours, making rotational forces overwhelmingly stronger than gravity. However, although the torus was discovered almost two decades ago, the mechanisms responsible for its continued existence remain elusive. Both simple analytic calculations and detailed numerical simulations predict that centrifugal forces should fling torus plasma away from its position in one or two Jovian rotation periods. The inferred lifetime is several orders of magnitude longer, and there is no evidence of catastrophic disruption. (Papers in the book edited by Dessler [1983] give a thorough overview of torus characteristics.)

A rapidly rotating plasma distribution is unstable to self-driven motions when heavier flux tubes can interchange with lighter ones farther from the rotation axis. Because plasma content \( \eta \) (the number of ions per unit magnetic flux) decreases outward from Io, the outer torus and extended plasma disc appear to be inherently unstable, assuming the average ion mass does not vary significantly. Indeed, although compositional evidence first pointed to Io as the source of plasma (V. M. Vasyliūnas, personal communication, 1997), the near coincidence of the satellite’s orbit with the maximum in \( \eta \) strongly corroborates that conclusion. Centrifugally driven interchange was the logical mode of bulk plasma transport [Richardson et al., 1980]. Interchange releases centrifugal potential energy that provides the kinetic energy necessary for motion, so the interchange rate is controlled by the radial gradient in \( \eta \), the angular frequency \( \Omega \), and distance to the rotation axis \( r \).

Electromagnetic coupling with the Jovian ionosphere dissipates energy there, so its Pedersen conductance \( \Sigma \) is important as well. Indeed, in the inner and middle magnetosphere of Jupiter the ionosphere is the dominant constraint determining the instability growth rate. For this situation, Siscoe and Summers [1981] calculated a
Figure 1. The top sketch depicts the interchange mechanism proposed by Siscoe and Summers [1981]. Arrows indicate the circulating motion whereby heavier and lighter flux tubes exchange places, after which the pattern dissolves and is replaced by a similar, though spatially uncorrelated one. The bottom frame shows contours of constant $\eta$ late in an RCM-J run [Yang et al., 1994]. After the initial displacement, heavy plasma continues to fall outward and eventually forms extended fingers. Both are in the equatorial plane.

characteristic interchange timescale for the torus that is independent of the size of the circulating structure (Figure 1, top). Using a linear perturbation analysis, Huang and Hill [1991] showed the initial growth rate to be essentially the reciprocal of Siscoe and Summers' [1981] timescale, though the instability is suppressed for convective structures of large azimuthal wavelength. This theoretical prediction was confirmed by the numerical simulations of Yang et al. [1992, 1994] using the RCM-J, a version of the Rice convection model adapted for
the Jovian magnetosphere. New RCM-J simulations provide the basis for the present paper and will be discussed in more detail below.

Siscoe and Summers combined their timescale with an assumed characteristic eddy size to construct an eddy diffusion coefficient for a Fokker-Planck treatment of radial transport. This approach presumes that net outward transport arises from the mutual interference of numerous small-scale, continually reforming fluctuations. In the RCM-J results the character of the convection was quite unlike the stochastic picture envisioned. Instead, initially small perturbations grew unhindered to form long, extended fingers reaching from the torus region to the simulation domain’s outer boundary (Figure 1, bottom). These results agree with several other theoretical treatments of interchange in the Jovian magnetosphere [Pontius et al., 1986; Pontius, 1987; Southwood and Kivelson, 1989], which suggested a picture at odds with the assumptions underlying the radial diffusion formalism. A flux tube with higher $\eta$ than its surroundings feels a net centrifugal buoyancy force that drives it outward. This outward motion takes a massive, high-$\eta$ flux tube through a medium in which the average plasma content declines, so the high-$\eta$ flux tube finds itself increasingly massive compared to its neighbors. The centrifugal buoyancy force thereby increases and outward motion keeps getting faster, in sharp contrast to the diffusion picture, with its random distribution of waltzing eddies that move out, overturn, and dissociate. An initially unstable distribution quickly falls apart in the RCM-J simulations.

The timescale estimated for a small eddy to overturn thus also characterizes the expected lifetime of the entire unstable distribution. The persistence of the torus therefore implies that the convection patterns arising in the RCM-J are somehow suppressed. This problem was first discussed by Siscoe et al. [1981] in the context of the so-called plasma ramp at the outer edge of the torus. Between 7 and 8 Jovian radii ($R_j$), the inward gradient in flux tube content is much larger than on either side. In the radial diffusion model with no local sinks, such a feature implies that interchange motions are inhibited, which requires a larger gradient to produce the same net outflow. They suggested that a pressure gradient in the hotter particles could provide an opposing, inward force to stabilize the system. While cold, dense plasma from Io predominantly responds to the outward centrifugal force, hotter plasma resists inward displacement because of the accompanying compression. Unfortunately, the hot particle distribution was not completely measured by Voyager because of instrumental limitations, and this hypothesis has not yet been decisively tested. Modeling efforts using the observed populations have been unable to reproduce the ramp structure in the colder particles [Summers and Siscoe, 1985; Summers et al., 1988]. Mauk et al. [1996] recently presented an extensive analysis of Voyager low-energy charged particle data that significantly extends the known range of particle distributions, though it is still incomplete. Those authors explicitly conclude that hot particles cannot explain the plasma ramp.

Given the difficulties in accurately determining the fluid properties of the energetic particle population from limited spacecraft measurements, the question remains somewhat open whether hot particles can provide a force sufficient to suppress interchange motions and impound the torus. The present paper considers an alternative mechanism whereby the developing instability is interrupted by a shear in the background rotational velocity. Plasma is consistently observed to lag convection by about 4% from just inside Io's orbit to about 1 $R_j$ farther out, a region we will refer to as the velocity trough. Brown [1994] measured the radial profile of azimuthal velocity directly from the Doppler shift in spectral observations of the torus. His observations are precise enough to determine the local rotation frequency in the torus at positions separated by as little as 1/8 $R_j$. There appears to be little variation with magnetic longitude or orbital phase of Io, although a significant dawn-dusk asymmetry is present. The cause of this departure from corotation has long been understood to be plasma mass addition from ionizing Ioenic neutrals [Pontius and Hill, 1982]. Differential rotation exerts a torque that transfers angular momentum to the newly added plasma. Although mass loading remains the ultimate reason for slippage, Brown's observations and the concentration of mass loading near Io itself [Marconi and Smyth, 1996] pointed to Jupiter's atmosphere as the true location of most of the observed lag [Huang and Hill, 1989; Pontius, 1995]. Differential rotation between the torus and the high-altitude neutral atmosphere does exist, but it is negligible compared to that between heights within and below the ionsphere. The reaction time of the atmosphere is very long, so if mass loading in the torus ceased altogether, the observed lag would persist for many months as the atmosphere was slowly brought up to speed.

Regardless of the theoretical reasons for the velocity trough, its existence and persistence are firmly established from observations. The fact that the local angular frequency $\Omega$ in the torus is a function of $L$ raises the following question: What is the effect of differential rotation in the background flow on the development of the interchange instability? Pontius [1997] found that differential rotation does not change the linear growth rate when the ionsphere is strong enough to dominate plasma dynamics, as occurs in the torus. However, the present studies reveal a secondary effect of the velocity shear that may eventually suppress the instability. As the problem is typically posed, the interchange rate calculated corresponds to an instantaneous convection pattern. However, rather than simply evolving under its own influence, that pattern will be sheared by differential motion in the background flow. Any radially extended perturbation is gradually tilted sideways, which changes the distribution of parallel currents incident on
the ionosphere and hence the electric field determining convection. The direct association of density perturbation and radial motion is thereby interrupted.

In the present paper we will not consider the details of mass loading and atmospheric dynamics that determine the angular frequency but simply take \( \Omega(L) \) from Brown's observations. The following section presents a simplified analytic model as a heuristic explanation of how a velocity shear could suppress the interchange instability. We then describe the RCM-J and show simulation results that support the general findings of the theoretical model. The final section contains a summary and our conclusions.

2. Analytic Example

To demonstrate how a velocity shear might affect the growth of a small perturbation, we turn to a simplified example. The complications imposed by cylindrical geometry and radial variations are removed to facilitate analytic solution, but these are not essential to the novel features arising in the more complicated simulations. Several detailed theoretical treatments of the interchange instability in a rotating magnetosphere are available [e.g., Southwood and Kivelson, 1989; Huang and Hill, 1991], but none appears adaptable for differential rotation.

In Cartesian coordinates \((x, y, z)\) with corresponding unit vectors \((\hat{e}_x, \hat{e}_y, \hat{e}_z)\), consider a uniform magnetic field \(\mathbf{B} = -B \hat{e}_z\) between two identical parallel-plane ionospheres at \(z = \pm H/2\), unbounded in the \(x\) and \(y\) directions and each having constant Pedersen conductance \(\Sigma\). To play the role of the centrifugal force, we impose a uniform external force \(\mathbf{F} = F \hat{e}_x\) proportional to particle mass. Flux tube content is generally defined by

\[
\eta \equiv \int \frac{n}{B} \, ds, \quad (1)
\]

where \(n\) is number density and integration is along field lines. Because \(\mathbf{F}\) is orthogonal to \(\mathbf{B}\), plasma is uniformly distributed along field lines, so \(\eta\) is simply \(nH/B\). The force drives a drift current normal to \(\mathbf{B}\) and \(\mathbf{F}\), and integrating along the magnetic field, the perpendicular current per unit length in the \(xy\) plane is \(J_\perp = \eta F \hat{e}_y\). Any variation of \(\eta\) with \(y\) makes \(J_\perp\) diverge, which drives parallel currents into the ionosphere. These must be balanced by ionospheric currents \(J_\parallel\) with equal and opposite divergence. Relating this model to the Jovian magnetodisk, \(\hat{e}_x\) and \(\hat{e}_y\) correspond respectively to the local radial and azimuthal directions.

Ionospheric Pedersen currents are driven by the electric field measured in a frame at rest with respect to the local neutral populations. However, we are interested in a differentially moving ionosphere that lacks a single, preferred reference frame. Using an arbitrary inertial frame, it is convenient to express the total electric field \(\mathbf{E}\) based on the steady and varying parts of the convection using the frozen-in flow condition. The first part equals \(-\mathbf{V}_o \times \mathbf{B}\), where \(\mathbf{V}_o\) includes the background flow representing the velocity trough and does not change during the calculation. The second \(\delta\mathbf{E}\) arises from the perturbations themselves and produces an evolving flow field \(\delta\mathbf{v}\) governed by \(\delta\mathbf{E} + \delta\mathbf{v} \times \mathbf{B} = 0\). This convection drives ionospheric currents \(J_\parallel = \Sigma \delta\mathbf{E}\), so that

\[
\nabla \cdot \delta\mathbf{E} = -\frac{\mathbf{F}}{2\Sigma} \frac{\partial \eta}{\partial y}. \quad (2)
\]

The factor of 2 arises to account for both ionospheres. The continuity equation is

\[
\frac{\partial \eta}{\partial t} + (\mathbf{V}_o + \delta\mathbf{v}) \cdot \nabla \eta = 0, \quad (3)
\]

in the absence of sources and sinks because the evolution of \(\eta\) is governed by the entire flow field.

To model growth from an initial state of unstable equilibrium, we choose a background plasma distribution \(\eta_0\) that depends only on \(x\) and has constant gradient \(\eta_\parallel < 0\). Upon applying an initial \(x\) displacement that is sinusoidal in \(y\), the perturbation in flux tube content is

\[
\delta\eta = -A\eta'_0 \sin ky. \quad (4)
\]

The amplitude \(A\) is independent of position but may depend on time. Equation (2) becomes

\[
\nabla \cdot \delta\mathbf{E} = \frac{A\eta'_0 F}{2\Sigma} \frac{\sin ky}{\partial y}. \quad (5)
\]

The right side is independent of \(x\), and integration in \(y\) gives \(\delta\mathbf{E}_y\) plus a term proportional to \(\partial_y \delta\mathbf{E}_x\). Because the parallel current density is independent of \(x\), the simplest solution of (5) has \(\partial_x = 0\); to keep the electrostatic potential bounded, \(\delta\mathbf{E}_x\) must therefore be zero. Having obtained \(\delta\mathbf{E}_y\), the frozen-in flow condition implies that

\[
\delta\mathbf{v} = -\frac{F}{2\Sigma B} A\eta'_0 \sin ky \hat{e}_x = \frac{\delta\eta}{\Sigma B} \hat{e}_x. \quad (6)
\]

This is the instantaneous flow field produced by a family of identical sinusoidal perturbations that are aligned in \(y\) for all values of \(x\) (Figure 2, top).

Consider a situation in which \(\mathbf{V}_o = 0\). By inspection, the perturbation plasma content (4) and flow (6) will satisfy the continuity equation at all times only if the amplitude \(A\) varies with time according to

\[
A(t) = A_0 \exp \left[ \frac{F|\eta'_0|}{2\Sigma B} t \right] = A_0 \exp \{\gamma t\}, \quad (7)
\]

where \(A_0 = \text{const}\) is the initial amplitude. Excepting geometric factors, this result agrees with the more rigorous treatments cited above. Ordinarily, \(\gamma\) is the linear growth rate for an infinitesimal perturbation, but in the special geometry adopted here, it describes the growth rate at all times.

Now let there be a constant velocity shear such that \(\mathbf{V}_o \propto x \hat{e}_y\) and a line initially extended in the \(x\) direction
will become tilted by angle \( \alpha \) at time \( t \):
\[
\tan \alpha = \frac{V_0 t}{x} = \omega_{\alpha} t,
\]
where \( 1/\omega_{\alpha} = x/V_0 \) is the time required for \( \alpha \) to grow from 0 to \( \pi/4 \). Even in the absence of a perturbation flow, the shear itself would displace contours of \( \eta \) relative to each other. If \( \delta v \) were zero, the continuity equation could be satisfied if the plasma perturbation were altered to read
\[
\delta \eta = -A \eta_0 \sin k (y - x \tan \alpha)
\]
with \( A \) a constant. Of course, a complete solution for nonzero \( \delta v \) requires considering the evolution produced by both the velocity shear and the perturbation flow. Remarkably, (9) does satisfy the full continuity equation for a properly chosen \( A \), as we will now show.

Let us calculate the instantaneous flow field produced by the shifted \( \delta \eta = \text{const} \) sinusoids described by (9). The divergence of the electric field is now
\[
\nabla \cdot \delta \mathbf{E} = \frac{FA \eta_0 \partial \sin k(y - x \tan \alpha)}{2\Sigma}.
\]
The right side is a function of both \( x \) and \( y \), but under a simple coordinate rotation
\[
\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}
\]
equation (10) becomes
\[
\nabla \cdot \delta \mathbf{E} = \frac{FA \eta_0}{2\Sigma} \left( \frac{\partial y'}{\partial y} \right) \frac{\partial \sin k'y'}{\partial y'}.
\]
This equation has translational symmetry in the \( x' \) di-

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure2}
\caption{The plane normal to \( B \) in the simplified rectilinear model. Solid lines are curves of constant \( \eta \) and dashes indicate lines of constant phase. Quantities along the sides indicate background quantities, while those between \( \eta \) contours indicate perturbation electric fields, currents, and flows.}
\end{figure}
rection and is periodic in $y'$ with wave number $k' \equiv k / \cos \alpha$. The resulting configuration of parallel currents is geometrically similar to the aligned case (5), and integration in $y'$ gives the flow field (Figure 2, bottom):

$$\delta v = \frac{F}{2 \sqrt{2} B} \eta_0 \cos \alpha \sin k' y' \hat{e}_x.$$  

Like the perturbation density (9), this flow is symmetric in $x'$, which means that the $\delta v \cdot \nabla \delta \eta$ term in the continuity equation (3) is identically zero. Simple substitution shows that (9) and (13) satisfy the continuity equation for all times if and only if the amplitude varies as

$$A(t) = A_0 \exp \left[ \frac{\gamma \alpha(t)}{\omega_\alpha} \right],$$  

where $A_0$ is the amplitude at $t = 0$ when $\delta \eta$ is aligned in $x$.

Within our simplified mathematical model, the solution obtained for uniformly sheared perturbations does not depend on any approximations and fully describes the system's evolution. At early times, $\alpha \approx \tan \alpha = \omega_\alpha t$, so the perturbation grows exponentially at the rate calculated for the unsheared case. However, rather than growing indefinitely, $A$ asymptotes to a finite value determined by the ratio of $\gamma$ to $\omega_\alpha$. If they are equal, $A$ is limited to $\exp(\pi/2) \approx 5$ times its initial value, while if $\gamma / \omega_\alpha < 2 \ln 2 / \pi = 0.44$, it never even doubles. For the Jovian velocity trough, observations described in the next section suggest a nominal peak value $\gamma / \omega_\alpha = O(1)$, though the exact value depends on the ionospheric conductivity adopted.

Although this example does not capture all the complexities of the physical situation, it does suggest how a velocity shear might stabilize an energetically unfavorable configuration. The next section employs a method that can treat more complicated geometries with spatial variations in both the velocity shear and the perturbation amplitude.

3. Numerical Simulations Using the RCM-J

The Rice convection model was originally developed to study convection in the inner regions of Earth's magnetosphere. Its logical foundation, supplied by Va-syl'kines [1970], depends on the flow speeds being much less than the Alfvén speed and timescales being much longer than the Alfvén-wave travel time. Just as in the example presented above, currents that diverge in the magnetosphere must close through the ionosphere, and solving Poisson's equation there gives the electric field. The convective flow determined by this electric field and other particle drifts is used to move the plasma, from which the new pattern is determined.

For Jupiter, the model was adapted to include the currents arising from the centrifugal force. Because Io-genic plasma is relatively cold, it remains confined near the equatorial plane, and the electric field provides the only significant transport of particles across magnetic field lines. The Jovian magnetic field is represented by a spin-aligned dipole with equipotential field lines, so the condition $\delta \mathbf{B} + \delta \mathbf{v} \times \mathbf{B} = 0$ imposes a one-to-one relation between field lines and ionospheric footprints. This reduces the problem to two dimensions on a surface intersecting all the field lines in the region of interest. While the computer code itself is written in ionospheric coordinates, results can be presented either there or by mapping to the equatorial plane. The smoothly varying physical plasma distribution is modeled as a discrete set of plateaus of constant $\eta$, analogous to a terraced hill. In the absence of sources and losses, $\eta$ is conserved by the flow, so convection simply moves the boundaries between adjacent plateaus. The RCM-J tracks a collection of test particles, each on one of the boundaries. Connecting neighboring points on a given boundary gives a contour approximating the smooth physical curve. These contours are then used to calculate current divergence. Details about the RCM-J are available in the work by Yang et al. [1994], (henceforth referred to as Paper 1) along with references to technical information about the original RCM.

We used the RCM-J to simulate the evolution of the torus from a specified initial plasma configuration with no local plasma source. For the present simulations, the RCM-J is initialized as described in section 3 of Paper 1. The magnetic field is a static, Jovicentric, spin-aligned dipole, and the ionospheric Pedersen conductance $\Sigma$ is uniform and constant. The magnetic $L$ parameter is simply the equatorial crossing distance of a field line divided by the Jovian radius $R_J = 71,400$ km and is related to surface latitude $\theta$ by $L \cos^2 \theta = 1$. For $L > 5.91$, the background plasma distribution is designed to represent the function

$$\eta_0 = 1.283 \times 10^{23} \text{Weber}^{-1} \left( \frac{L}{5.91} \right)^{-2.2}$$  

This distribution is based on an analysis of Voyager plasma science (PSS) data by Bagenal et al. [1986]. A refined analysis by Bagenal [1994] differs slightly, but we use this expression to facilitate comparisons with the earlier results of Paper 1. A collection of 10 distinct contours represents $\eta$ in the manner described above, as shown in Figure 1 of Paper 1. Inside the innermost contour, initially at $L = 5.37$, the plasma content falls to zero. The initial perturbation is sinusoidal with azimuthal wavenumber $m = 8$, so solutions are periodic over 45º of longitude. The simulation domain is defined by $4 \leq L \leq 12$, corresponding to ionospheric latitudes $60^\circ \leq \theta \leq 73.2^\circ$, and azimuthal coordinate $0 \leq \phi \leq 45^\circ$. One difference from Paper 1 is that the initial perturbation is 40 times larger, so a given con-
tour is displaced in $L$ by approximately 0.04 from its unperturbed value. Growth is extremely slow in the early stages, so this change eliminates a great deal of computer time during which little of interest was happening in the simulations of Paper 1.

The novel feature of the present numerical experiments is the addition of an unperturbable background flow $V_0$ in the magnetosphere. To describe the velocity profile in the equatorial plane as measured in a frame corotating with Jupiter, we construct the following analytic approximation of Brown's [1994] observations:

$$V_0(L) = -\frac{\sqrt{2} \sigma v_{\text{max}}}{\Delta L} \exp \left[ - \left( \frac{L - L_o}{\Delta L} \right)^2 \right] \times \max [0, L - L_o] \; \hat{e}_\phi, \quad (16)$$

where $v_{\text{max}}$ is 4 km/s (Figure 3). The width of the velocity trough is $\Delta L = 1.25$, and its inner boundary is at $L_o = 4.78$. M. E. Brown (personal communication, 1996) has cautioned that his observations probably underestimate how quickly the velocity perturbation decreases in outer regions of the torus because of line-of-sight integration. Hence the physical velocity shear is likely to be even steeper than implied by this expression.

As a technical matter, the RCM-J does not directly follow ionospheric neutral populations as it does ions in the magnetosphere. Rather, atmospheric motions are incorporated by specifying the electric field required to move ions with those winds according to $E + V_0 \times B = 0$. The resulting net convection pattern reflects the influence of the winds correctly. In the present case, we added a term to Poisson's equation proportional to the divergence of the ionospheric electric field associated with (16). The derivation is straightforward, but as the dipole field mapping makes the final expression quite cumbersome, we do not include it here.

Unlike the analytic example, the velocity profile (16) does not have a constant shear, which suggests we should anticipate consequences caused by its finite extent. An initially radial, infinitesimal line segment will be rotated by the shear in $V_0 = R_j L \Delta \Omega$, so a radially dependent angle $\alpha$ and associated shear rate $\omega_\alpha$ can be defined for this situation as

$$\tan \alpha \equiv \frac{d\Omega}{dL} t = \omega_\alpha t. \quad (17)$$

Here, $1/\omega_\alpha$ is the time required for $\alpha$ at position $L$ in the equatorial plane to increase from zero to $\pi/4$. Because of dipole magnetic mapping, the ionospheric image of this angle has twice as large a tangent. Figure 4 plots $\omega_\alpha$ normalized to Jupiter's rotation frequency $\Omega_j = 1.74 \times 10^{-4} \text{s}^{-1}$ as a function of latitude in the ionosphere where the RCM-J results will be displayed. The shear is largest between roughly $66^\circ$ and $67^\circ$ but remains substantial for at least half a degree on either side of that region. There is also a large shear inside of Io's orbit, but the gradient of $\eta$ is stable against the interchange instability there.

In the rectilinear example, the saturation amplitude
depends on the magnitude of $\omega_\alpha$ relative to $\gamma$, the growth rate in an unsheared configuration. From Huang and Hill [1991], the centrifugal growth rate for rigid rotation in a dipole field is

$$\gamma = \frac{m_i \eta_0 \Omega_J^2 R_J L^4}{2 \Sigma B_J} \approx 0.5 \Omega_J \left( \frac{1 \text{ mho}}{\Sigma} \right), \quad (18)$$

using (15) with an average ion mass $m_i = 20$ amu and setting $L = 6$. This expression is appropriate for torus perturbations of azimuthal wavelength much smaller than the length scale for radial variations in $\eta_0$. Although no rigorous analytic treatment of differential rotation in a dipole field is available, we speculate that $\gamma/\omega_\alpha$ will be the critical parameter in this situation as well. If the ratio is large everywhere, the instability should be essentially unaffected by the shear. However, if it is small throughout a region much larger than the initial perturbation amplitude, then neighboring contours of constant $\eta$ will be shifted relative to one another before they can grow substantially. The resulting phase decorrelation should impede outward motion much as in the analytic example.

The velocity shear under consideration extends for a degree or two in latitude, so instability can be suppressed only if growth saturates well before a perturbation reaches this size. A perturbation that can extend outside the shear region before becoming significantly distorted should continue growing without further interference. The initial perturbations in the present simulations vary in ionospheric latitude by slightly less than a tenth of a degree. While this is sufficiently smaller than the width of the shear, a fairly modest growth would violate that condition. Hence, stabilizing the system against a perturbation of this magnitude requires that its growth be very rapidly halted. If the saturation amplitude is an exponential function of $\gamma/\omega_\alpha$, as suggested by (14), stability requires that $\gamma$ be much smaller than the maximum value of $\omega_\alpha$.

For the first set of trials (not shown) the Pedersen conductance was set at $\Sigma = 1$ mho, the value used in Paper 1. Equation (18) predicts $\gamma = 0.5 \Omega_J$, so the minimum value of $\gamma/\omega_\alpha$ is approximately 2. This suggests that a $0.1^\circ$ initial perturbation should grow much larger than the shear region and not be greatly influenced by it. Indeed, the differences between simulations with and without a velocity shear were not dramatic. In both cases the system evolved similarly to the previous trials, with growth rates in agreement with the linear prediction (18). The initially small perturbations grew rapidly and formed long fingers, with heavier flux tubes quickly reaching the outer boundary. The fingers were slightly skewed near their bases in the velocity trough, but the overall instability was essentially unaffected.

Because the velocity profile is constrained by observation, significant differences due to a velocity shear require reducing the unsheared growth rate. Beyond the mechanism considered here, there are several reasons
that the growth rate may be substantially slower. First, the ring-current will probably suppress interchange to some degree as proposed by Slasoe et al. [1981]. Although these hotter particles are not included in the present simulations, using a larger Pedersen conductance can provide a proxy for their influence. Second, the physical Pedersen conductance itself is not well known, and the nominal value $\Sigma = 1$ mho is in the middle of the range 0.1 to 10 mho calculated from ionospheric models [Hill et al., 1981]. An indirect indication of the ratio of $\Sigma$ to the mass transport rate is available from in situ observations of the average angular velocity in the plasma sheet [Hill, 1979], and the generally accepted transport rate gives $\Sigma \approx 0.4$ mho. However, Hill's calculation actually depends on an effective conductivity that is reduced from the actual value depending on the atmospheric interaction [Huang and Hill, 1989]. One consequence of the flywheel model is

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{RCM-J results, showing contours of constant flux tube content $\eta$ in the Jovian ionosphere, with the pole toward the top of the figure. The 90° range of longitude displayed includes two periods of the periodic simulations. Frames in the right column are from simulations including the velocity shear.}
\end{figure}
that the true conductivity must be at least 10 times larger than the effective value [Pontius, 1995].

Given the preceding considerations, our second set of trials started from the same conditions but employed a value $\Sigma = 8$ mho, so that $\gamma/\omega_p$ is less than $1/4$ over a fairly extensive band of latitude. The unsheared control simulation evolved as before, only 8 times more slowly. However, the simulation including the velocity shear behaved very differently. Figure 5 shows $\eta$ contours at various times in the simulations. In the top frames at 50 hours, the perturbations are still close to the linear stage of development. In the unsheared case on the left, the $\eta$ contours all have similar sinusoidal shapes and are in phase with each other. Those at lower latitudes have smaller amplitudes due to the restraining influence of the innermost contour, which was not
perturbed initially. In the corresponding sheared solution on the right, the contours at high latitude are still roughly sinusoidal, though their amplitudes are somewhat reduced. However, moving to lower latitudes, the relative azimuthal displacement of neighboring contours due to the shear becomes increasingly evident. In the heart of the sheared region, a pair of points separated by one degree in latitude have drifted relative to each other by as much as 45° in longitude. The perturbations are manifestly out of phase, and their amplitudes are much smaller than in the unsheared trial, as expected.

Turning to the middle frames at 90 hours, the unsheared solution shows rapid growth as the unrestrained centrifugal force drives heavier plasma outward, that is, to higher latitudes. In the sheared case, the four outermost contours are still growing because they are essentially outside the velocity trough, though their amplitudes are half as large as they would have been. For the remaining contours in the strongest part of the shear, only minor changes in amplitudes compared to the top frame are discernible. The curves are increasingly distorted, but there is very little expansion in latitude. This trend continues in the bottom frames at 110 hours, and some contours even develop sharp zigzags as adjacent points are drawn in opposite directions past one another. In contrast, the unsheared simulation has passed the limit where the solutions are physically meaningful because plasma contours have reached the outer boundary of the simulation domain.

It is interesting that the velocity shear manages to stabilize the inner contours despite the proximity of the outer contours, which are still quite unstable. This means that the fringing electric fields associated with the growing perturbations are shielded from lower latitudes. Figure 6 shows the η contours at 190 hours together with the electric equipotentials at that time. The electric potentials corresponding to the velocity trough itself have been suppressed. The outermost plasma contours have grown from large perturbations into well formed, outward moving fingers. The inner contours show no tendency to continue moving radially, and the relative drifts in azimuth are responsible for most of the further evolution. This is evident in the plot of equipotentials, which are also instantaneous streamlines for cold particles. The perturbation electric field is essentially zero inside of 67°, and equipotentials associated with the growing fingers are excluded from lower latitudes.

4. Conclusions

The RCM-J simulations support our hypothesis that under suitable conditions a velocity shear can stabilize an otherwise unstable distribution. This stabilization can occur even if the velocity shear is not infinite in extent, so long as the perturbations remain inside the region with substantial shear. For the analytic example, the amplitude of an initially aligned perturbation can expand by a factor that depends exponentially on the ratio of the unsheared growth rate γ to the shear rate ωs. A similar result appears to hold in the RCM-J simulations, with an additional restriction on the initial perturbation amplitude A0. The range ΔL of ΔL values over which a significant differential rotation is distributed sets an immediate upper limit on A0, but the saturation amplitude must also be much less than ΔL. If the net differential rotation is ΔΩ, then ωs can be approximated by LΔΩ/ΔL. Therefore, given γ, a relation for a dipole field analogous to (14) would set an upper limit on A0.

Comparing the simulation results (Figures 5 and 6) to the profile of ωs (Figure 4) shows that the boundary between stable and unstable regions falls where ωs decreases below some critical value. In the present tests using the velocity profile (16), this position is somewhat inside where the ramp begins, near L = 7. In retrospect, the formula we used to represent the trough is probably too confined and peaks at too low an L value. We are currently working on enhancing the RCM-J to include a plasma source, and future simulations will incorporate what is probably a more appropriate velocity trough. Nevertheless, the present simulations clearly demonstrate that the key observational parameter is ωs, and that its value relative to the unsheared growth rate γ determines the magnitude of perturbations that can be suppressed.

This mechanism has further consequences for the energetic particle populations. Constraining interchange would allow hot particles more time to deplete via ionospheric precipitation during their gradual migration inward. Their radial gradient is thereby increased, which further slows the unsheared growth rate and strengthens the velocity shear impoundment. We have not discussed the velocity shear at the inner edge of the velocity trough because the plasma mass distribution there is interchangeable stable. However, even where the distribution is stable, forced interchange motions may be driven by ionospheric winds [Brice and McDonough, 1973], and the velocity shear may also be important in suppressing that mode of transport. Once again, the energetic particle population can be expected to develop a more pronounced gradient as its inward transport is slowed.

To summarize, we propose that the ramp begins where the background velocity shear can no longer suppress the development of the centrifugal interchange instability for perturbations of typical magnitude. We demonstrated the potential of this mechanism using a sinusoidal disturbance and found a fairly distinct boundary between stable and unstable regions. This boundary would undoubtedly be broader for a range of initial perturbation sizes, as the local value of ωs determines the local limit on stability. Mechanisms that produce initial perturbations are beyond the scope of the present discussion and require considering the particulars of mass addition as well as the possibility of external forcing and occasional triggering.
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References


